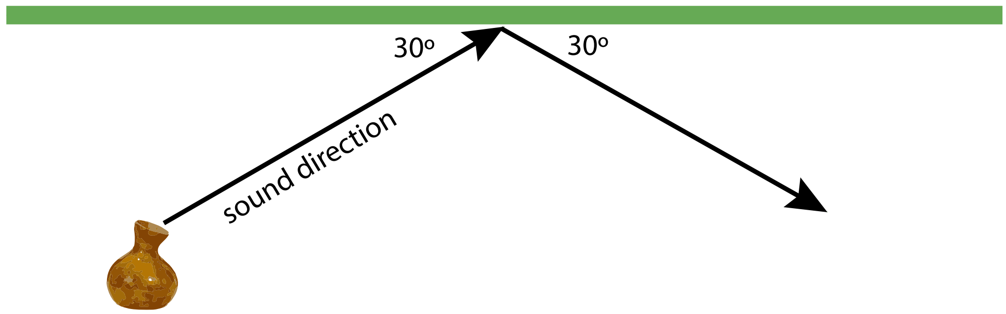
Unit 5: Measurement Geometry

Activity 5.1 - Sound reflections I

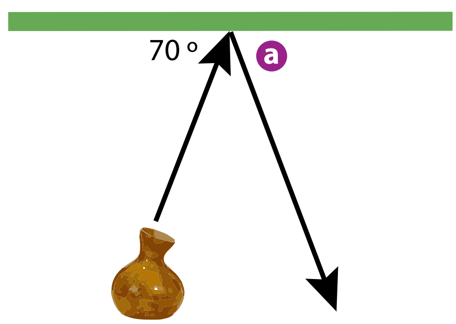
Sound reflects off of hard surfaces. The angle that sound approaches a surface is the same as the angle that it leaves the surface when it is reflected. For example, if you play a ipu and the sound wave hits a wall at 30°, then it will reflect away at a 30° angle and continue on its way. This is called the Law of Reflection.



Side note: This is a simplification of how waves move. Remember that waves actually spread out in many directions instead of staying together and moving in one line. Let's just keep it simple for now. Later, we'll look at sound waves moving in different directions.

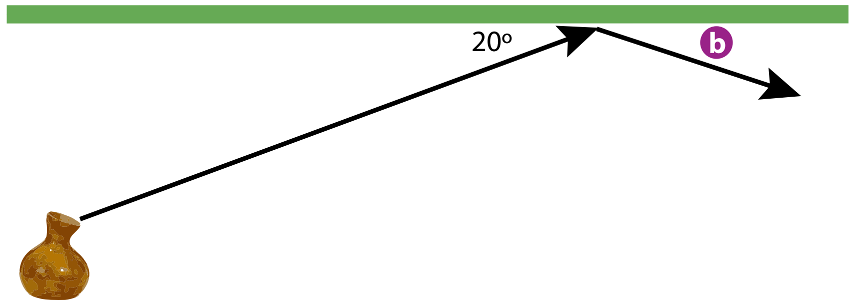
1. What are the measures of the following angles?

a. Angle a.



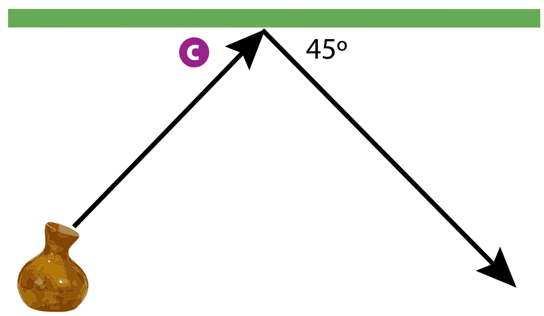
The Law of Reflections state that angle that sound approaches a surface is the same as the angle that it leaves the surface. So $a = 70^\circ$

b. Angle b.



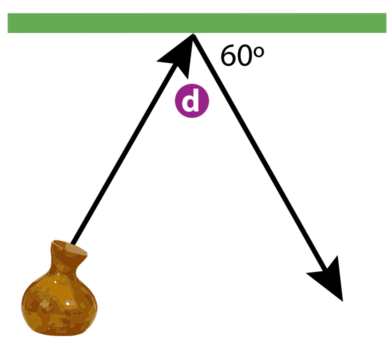
$b = 20°$

c. Angle c.



$c = 45°$

d. Angle d.



The angle of the sound leaving the wall is given to be $60°$ so by the Law of Reflection, the angle of the sound approaching the wall is also $60°$. All three angles are supplementary, meaning that they sum up to 180° to make up the straight wall. So $d = 60^\circ$.

$$\begin{aligned} 60^\circ + 60^\circ + d &= 180^\circ \\ 120^\circ + d &= 180^\circ \\ -120^\circ & \quad -120^\circ \\ d &= 60^\circ \end{aligned}$$

2. This is related to another law called Snell’s Law. Look on the internet to see what Snell’s Law is about.

a. What are some of the many things that Snell’s Law talks about? It can get complicated so be sure to work together and share your ideas with your classmates.

This is an exercise on researching. We do not expect the students to fully grasp the entirety of Snell’s Law. We just want the students to be resourceful in learning something that they may not have heard of previously. Snell’s Law discusses how light, waves, and other moving things change direction when their environment changes. For example, when light goes from air to water, when sound reaches a hard ceiling, when a pool ball hits a wall, etc.

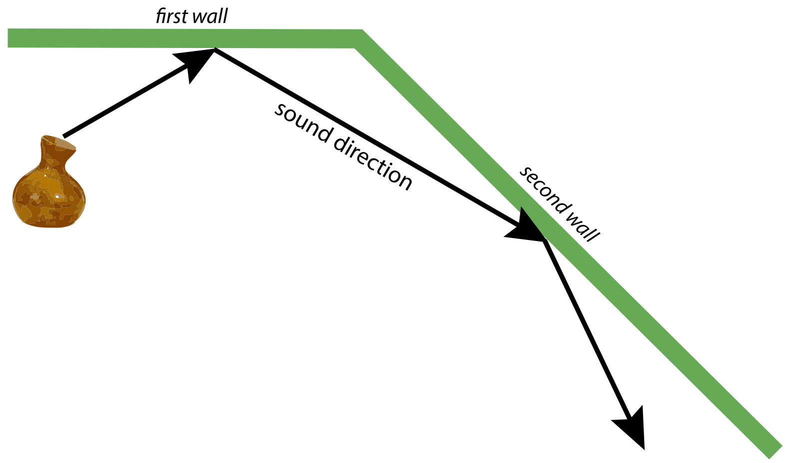
b. Where do you see Snell’s Law in real life? Share your ideas in your classroom or on the online comment section.

Some examples were mentioned in the previous teacher notes. The Hawaiians had a great intuition of how Snell’s Law applies to the ocean and used it to help with fishing (throwing nets, spears, etc.).

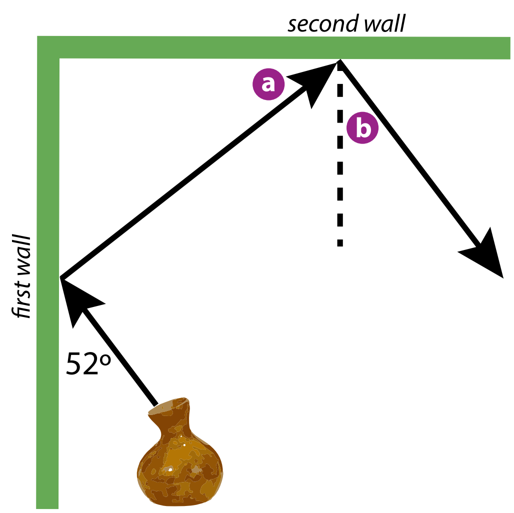
Activity 5.2 - Sound reflections II

Recall that the Law of Reflection says that when sound approaches a hard surface and is reflected away, the angle that it approaches the surface is equal to the angle that it leaves.

1. When sound wave moves toward a corner with two walls, the sound usually reflects off of both walls before going off in some direction in the room.



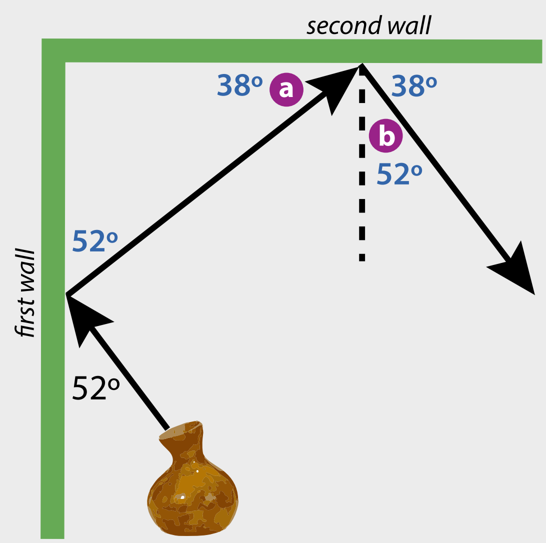
Let's take a look at what happens when the two walls are perpendicular to each other.



a. Find the measure of angle a. Explain your work, including when you needed to use the Law of Reflection or a math theorem.

b. The dashed line is parallel to first wall. Find the measure of angle b. Explain your work.

1. The angle of the sound leaving the first wall is $52°$ by the Law of Reflection.
2. That angle, the right angle corner, and angle a are supplementary and add up to $180°$ by the Triangle Sum Theorem. So $a = 38°$.
3. Since sound approaches the second wall at $38°$, it leaves the wall at $38°$ by the Law of Reflection.
4. Since the dash line is *parallel* to the first wall, and the first wall is *perpendicular* to the second, we know that the dash line is *perpendicular* to the second wall. We can prove this with the parallel lines theorems. Most student might not have the rigor (yet) to see that this must be proven. Feel free to point this out if you feel that your students are ready for the rigor.
5. That angle and angle b make up the angle between the second wall and the dashed line. So they are complementary. Thus $b = 52°$.



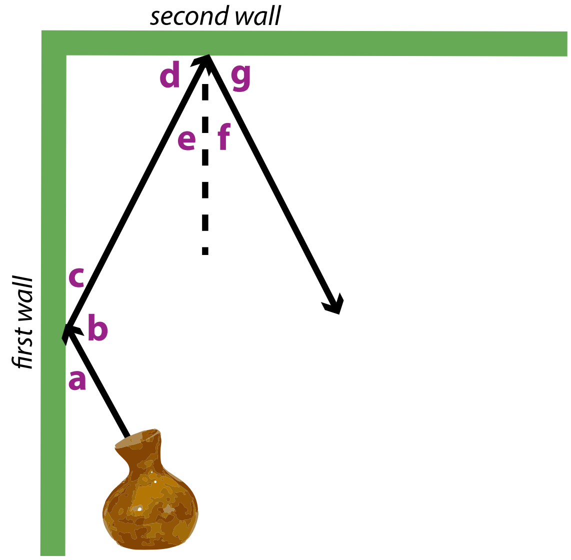
c. What can you say about the *direction of the sound wave approaching the first wall* compared to the *direction of the sound wave leaving the second wall*?

Using parallel line theorems, we see that sound wave going into perpendicular corners will leave in a parallel and opposite direction. So it will go back in the same direction it came.

d. When designing a *hale* (house or building) for listening to music, architects usually avoid having perpendicular walls in their designs. Why do you think that is?

When sound waves approach a corner with perpendicular walls, they reflect and leave the corner in the same direction that they came. In a hale with perpendicular walls, a musician, radio, or other sound source will experience an echo as a result. When sound waves are created, they will spread to the corners and come back. This echo can really ruin a musical experience.

2. Here's another 90° corner. The dashed line is parallel to the first wall. Angles a and c are congruent. Angles d and e are complementary.



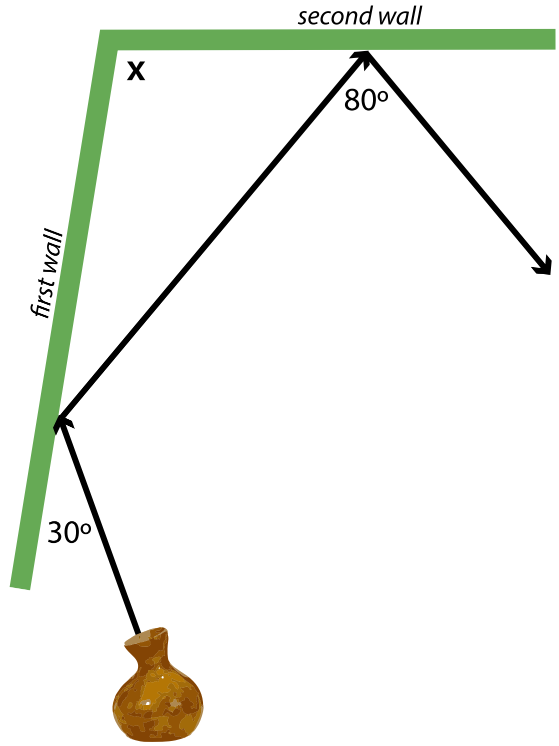
a. What are four other pairs of congruent angles? Write as pairs, for example a & c.

Angles a, c, e, and f are all congruent. Any four pairs from this list are acceptable answers. There are 6 possible answers, including the one given. We recommend having the students share how they came to their conclusions.

b. What are four other pairs of complementary angles? Write as pairs, for example d & e.

Angles d and e are complementary to every angle mentioned in part a. So any combination of d or e with a, c, e, or f are acceptable answers. There are 8 possible answers. Again, we recommend having the students share how they came to their conclusions.

3. Here's a corner that is *not* at a right angle.



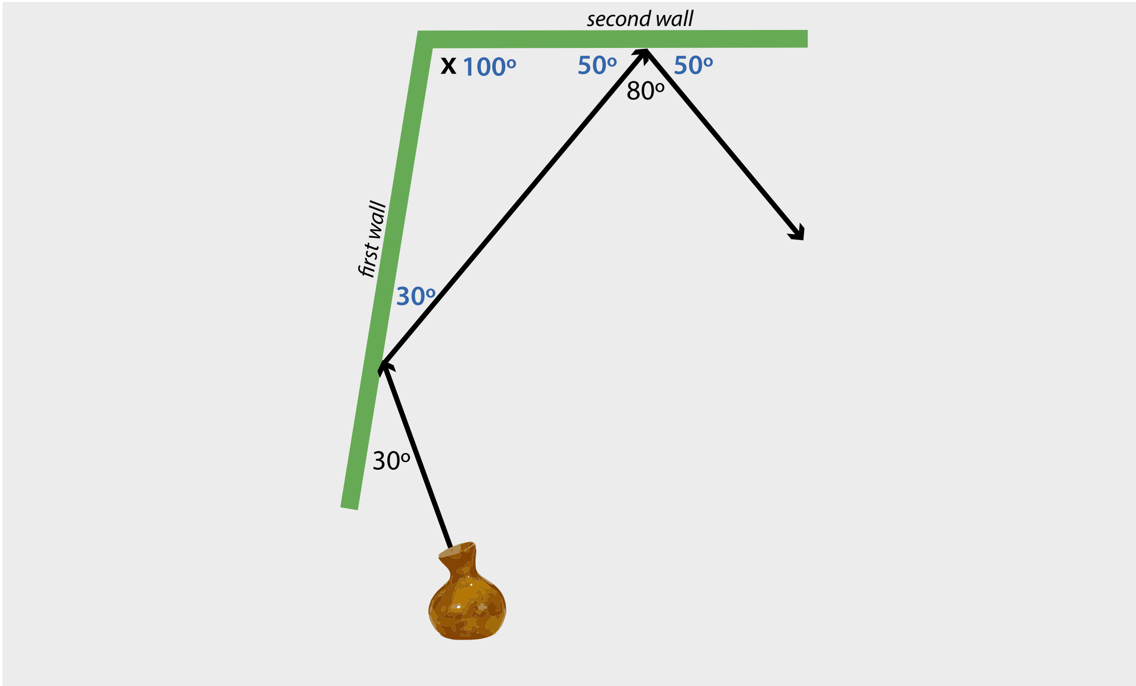
a. What is the measure of the angle of the corner, x?

1. The sound wave leaves the first wall at $30°$ by the Law of Reflection.

2. Then the sound wave approaches and leaves the second wall at the same angle by the Law of Reflection. Let's call this angle z. So z is the angle that the sound wave approaches the wall, z is also the angle that it leaves the wall, and $80°$ is the angle in between. These three angles are supplementary and sum up to $180°$. So z, the angle that the sound approaches the wall, is $50°$.

$$\begin{aligned} z+z+80° &= 180° \\ 2z+80° &= 180° \\ -80° & \quad -80° \\ 2z &= 100° \\ \div 2 & \quad \div 2 \\ z &= 50° \end{aligned}$$

3. Now we have that $30°$, $50°$, and x make up a triangle in the corner. We use the Triangle Sum Theorem to see that these angles add up to $180°$ so $x = 100°$.



b. Think about the *direction of the sound wave approaching the first wall* compared to the *direction of the sound wave leaving the second wall*. How does this compare to the perpendicular walls in parts 1 and 2?

Since the walls are not perpendicular, the sound wave that enters the corner is not parallel to the sound wave that leaves the corner. Since the sound waves have their directions turned to other places in the room, we don't have as big of a problem with a sound bouncing back and forth in one place, creating a weird series of repeating echoes.

Activity 5.3 - Reverberations

For this activity, use 340 m/s for the speed of sound. You may also need to use the Law of Reflection at some point.

Sound waves spread out in many directions. Sometimes we need to understand the distances of things around us and how that distance affects sound.

Imagine that your hula class takes place in a valley. When kumu plays an ipu, some of the sound will go straight to you so you hear it right away. Some of the sound will actually go away from you toward mauka (the mountains). Then the sound waves will reflect off the mountains and come back to you. You might hear the reflected sound right away if the mountains are close or you might hear the sound a couple of seconds later in an echo if the mountains are far away.

1. Suppose that you're at the coordinates (0 m, 0 m), your kumu is at (40.8 m, 54.4 m), and a mountain is at (224.4 m, 299.2 m). You may use a calculator for parts a-g.

a. How far away is your kumu from you?

We can use the distance formula, which comes from the Pythagorean Theorem, to find the distance between you and kumu. This distance is 68 meters.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(40.8 - 0)^2 + (54.4 - 0)^2} \\ &= \sqrt{40.8^2 + 54.4^2} \\ &= \sqrt{1664.64 + 2959.36} \\ &= \sqrt{4624} \\ &= 68 \end{aligned}$$

b. If kumu plays the *pū* (conch shell), how many seconds later will you first hear the sound of the pū? Round to three decimal places if needed.

We need to find the time it takes for sound to travel from the kumu to you. We can solve the system of linear equations $y = (340 \text{ m/s})t$ and $y = 68$ m to find the answer. This is equivalent to solving for t in 68 m $= (340 \text{ m/s})t$. From this, we see that it takes 0.2 seconds to first hear the sound.

$$\begin{aligned} (340 \text{ m/s})t &= 68 \text{ m} \\ \div(340 \text{ m/s}) & \quad \div(340 \text{ m/s}) \\ t &= 0.2 \text{ s} \end{aligned}$$

c. How far away are the mountains from the kumu?

We use the distance formula again to show that the kumu 306 meters away from the mountains

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(224.4-40.8)^2+(299.2-54.4)^2} \\ &= \sqrt{183.6^2+244.8^2} \\ &= \sqrt{33709+59927} \\ &= \sqrt{93636} \\ &= 306 \end{aligned}$$

d. How far away are the mountains from you?

The mountains are 374 meters away from you.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(224.4-0)^2+(299.2-0)^2} \\ &= \sqrt{224.4^2+299.2^2} \\ &= \sqrt{50355+89521} \\ &= \sqrt{139876} \\ &= 374 \end{aligned}$$

e. How far must the sound of the pū travel from the kumu to the mountains and then to you?

The path from the kumu to the mountains to you is 680 meters.

$$306 \text{ m} + 374 \text{ m} = 680 \text{ m}$$

f. How how many seconds after the kumu plays the pū, will you hear its sound after it reflects off of the mountains? Round to three decimal places if needed.

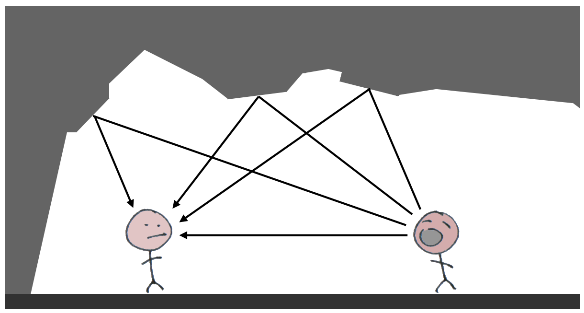
It takes 2 seconds for sound to travel from kumu to the mountains to you.

$$\begin{aligned} (340 \text{ m/s})t &= 680 \text{ m} \\ \div(340 \text{ m/s}) & \quad \div(340 \text{ m/s}) \\ t &= 2 \text{ s} \end{aligned}$$

g. How much time passes between when you first hear the sound and when you hear its echo from the mountains?

The first sound is heard 0.2 seconds after the kumu plays the pū. The sound is heard again 2 seconds after kumu plays the pū and the sound reflects from the mountains. The difference between these two times is 1.8 seconds.

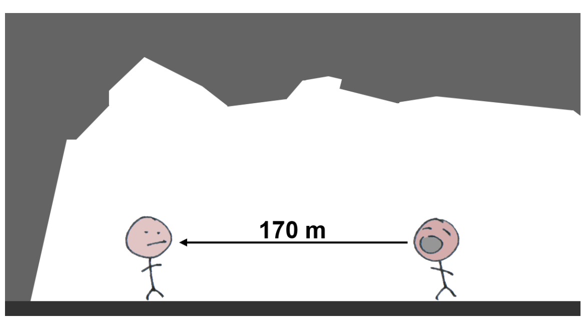
2. If you hear a sound once, then it travels far away, reflects, and you hear it a second time, this is called an *echo*. Since sound spreads in many directions, it is more common to have sounds bouncing all over the place before reaching your ears. So we don't just hear a clear sound twice. Instead we hear a sound many times stretched out over a short duration. This is called *reverberation*. Here's a picture of sound bouncing all over the *ana* (cave) before reaching the listener.



The first sound this listener hears is the sound that went straight to him/her. The last sound is the sound that bounced around the *ana* the furthest before reaching the listener. Roughly speaking, *reverberation time* is the time between when you hear the first sound and when most of the sound goes away.

Suppose that someone is singing 170 m away.

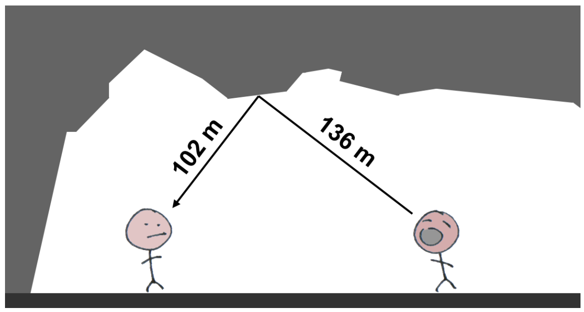
a. How long after the singer sings, does it take for the sound to first reach you? Round to three decimal places.



It takes 0.5 seconds for sound to travel straight from the singer to you.

$$\begin{aligned} (340\text{ m/s})t &= 170\text{ m} \\ \div(340\text{ m/s}) & \quad \div(340\text{ m/s}) \\ t &= 0.5\text{ s} \end{aligned}$$

b. Some of the sound does not go straight towards you, but instead bounces around first. If the slowest and last sound traveled 136 m to the ceiling before reflecting another 102 m to you, how long did it take for this sound to reach you? Round to three decimal places.



It takes 0.4 seconds for sound to travel from the singer to the ceiling and 0.3 seconds for sound to travel from the ceiling to you. This is a total of 0.7 seeconds.

$$\begin{aligned} (340\text{ m/s})t &= 136\text{ m} \\ \div(340\text{ m/s}) & \quad \div(340\text{ m/s}) \\ t &= 0.4\text{ s} \end{aligned}$$

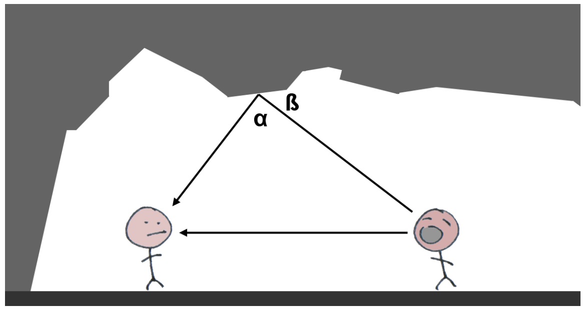
$$\begin{aligned} (340\text{ m/s})t &= 102\text{ m} \\ \div(340\text{ m/s}) & \quad \div(340\text{ m/s}) \\ t &= 0.3\text{ s} \end{aligned}$$

c. Suppose that it's silent after the last sound wave in Part 2b reaches you. What is the reverberation time?

The reverberation time is 0.2 seconds.

$$0.7\text{ s} - 0.5\text{ s} = 0.2\text{ s}$$

d. To control reverberation, we have to carefully design the angles and shape of the *ana* or where ever you like to listen to music. What is the measure of angle α? Hint: look at how the lines form a triangle. On each side of the triangle, write down how long it took sound to travel that distance.



The first and last sound waves form a triangle. If we write on the sides of the triangle, the time it takes to travel that part, then we have a triangle with sides 0.3, 0.4, 0.5. This is similar triangle (scale factor 0.1) to the special 3, 4, 5 right triangle. So the angle α is 90°.

e. Use your answer to Part d and the Law of Reflection to figure out the measure of angle β.

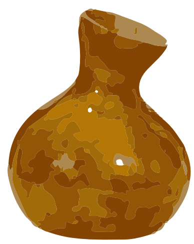
α, β, and a third angle are supplementary and make up the ceiling. By the Law of Reflection, β and the third angle are equal. So the angle β is 45°.

3. Think about times where you've been in a place with a lot or very little reverberation or echo. Describe the environment. What do you think it is about those environments that give it a lot or little reverberation and echo? Share your ideas in your classroom on the online comment section .

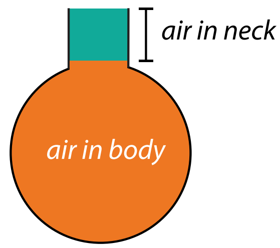
One place where I hear a lot of reverberation is in a empty hallway. This probably due to having walls close together and meeting at right angles. (Recall that we talked about perpendicular walls in Activity 5.2.) In some caves, reverberation can also be heard. In large valleys and canyons we can hear echoes. The large "walls" of a canyon and valley reflect a lot of sound and are usually far away so that there is a significant time difference between echoes. In an auditorium or concert, there usually is a pleasant amount of reverberation—not too long. These places have walls and ceilings specially designed with angles that scatter sound waves in just the right way. A room with a lot of carpeting and curtains has very little reverberation and echo. The soft material in this room absorb sound waves instead of reflecting them.

Activity 5.4 - Helmholtz Resonator

We know how the strings of a ʻukulele make sound waves. What about an ipu? A ipu works like a Helmholtz resonator, which is any instrument that is made up of a narrow tube connected to a larger hollow body. For example, a milk jug would make a great Helmholtz resonator.

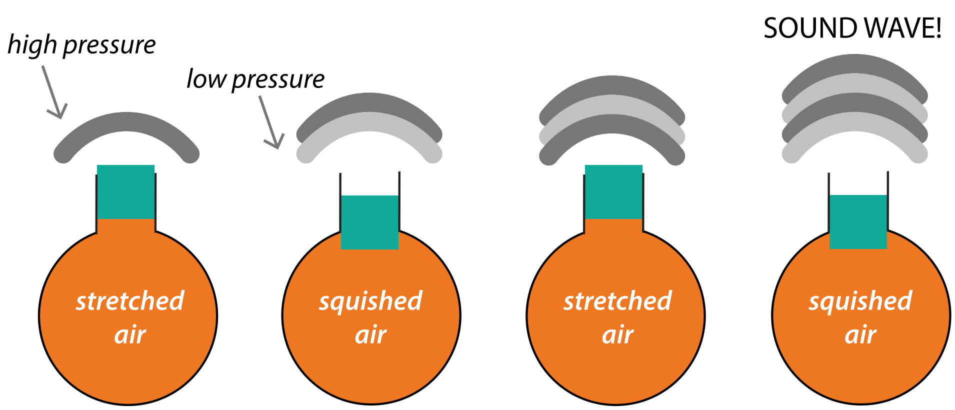


ipu



Helmholtz resonator

When you hit or blow across a Helmholtz resonator, the air in the neck lightly bounces in and out of the instrument. The air in the neck bounces because the air in the body can be squished and stretched. As the air in the neck bounces, it creates sound waves outside of the instrument.



The air in the NECK moves up and down, while air in the BODY stretches and squishes. The moving air in the NECK makes a sound wave outside!

1. Suppose that you're playing several ipu in a room. They all have the same neck, but the volume of their body is V. Then the frequency that the ipu make is $f = k\sqrt{1/V}$, where k is a constant positive number.

a. If you increase the volume of the ipu body, will you get a higher or lower sound? How can you tell by looking at the formula for frequency?

If you increase the volume V, then the denominator of the equation increases. Fractions get smaller when you increase the denominator so f gets smaller. So the sound gets lower if you increase the volume of the ipu body.

b. Rank the following ipu from *lowest* sound to *highest*. Everything expect the volumes of their bodies are the same. Assume that the body is a perfect sphere.
Ipu 1 has body radius 5 in.
Ipu 2 has body volume 900 in^3.
Ipu 3 has body diameter 13 in.
Ipu 4 has circumference of 8π inches at the widest part of the body.

We need to order each ipu by volume. Some students will notice that volume increases with radius so it is equivalent to order these ipu by radii. They might also notice that ordering by radius is easier for all but the second ipu.

Ipu	radius (in)	volume (in^3)
Ipu 1	5	523.599
Ipu 2	5.989	900
Ipu 3	6.5	1150.347
Ipu 4	4	268.083

Ranking from lowest to highest sound is the same as ranking from largest to smallest volume. So from low to high, we have Ipu 3, Ipu 2, Ipu 1, and Ipu 4.

c. What other objects (musical instruments or not), can be a Helmholtz resonator? Explain why and share your ideas with your classmates or on the online comment section.

In your chemistry and physics classes, you'll find a lot of examples of Helmholtz resonators. An Erlenmeyer flask and a round bottom flask are examples. At home you can find a 1-gallon milk jug and a 2-liter soda bottle that are also Helmholtz resonators. They are Helmholtz resonators because the air in the neck is in a cylindrical shape and much narrower compared to the body.

Something like a drinking straw that is closed on one end would not be a Helmholtz resonator. This is because there is no clear difference between the air in the body and the air in the neck.