

Unit 3: Solving Equations and Systems of Equations



Activity 3.1 - Stage Setup I

There are many different kinds of microphones. Choosing the right one depends mostly on what and where are you recording. If you want to record a sound coming from one direction and block out sounds from all other directions, then you might want to use a unidirectional microphone. These microphones clearly record what's right in front of it and ignores the stuff that's not in front of it.

Let's help set up the stage for a Hawaiian Choir of nine singers. On the y-axis are our three unidirectional microphones at the following points.

Mic A: (0, 4)

Mic B: (0, 8)

Mic C: (0, 13)

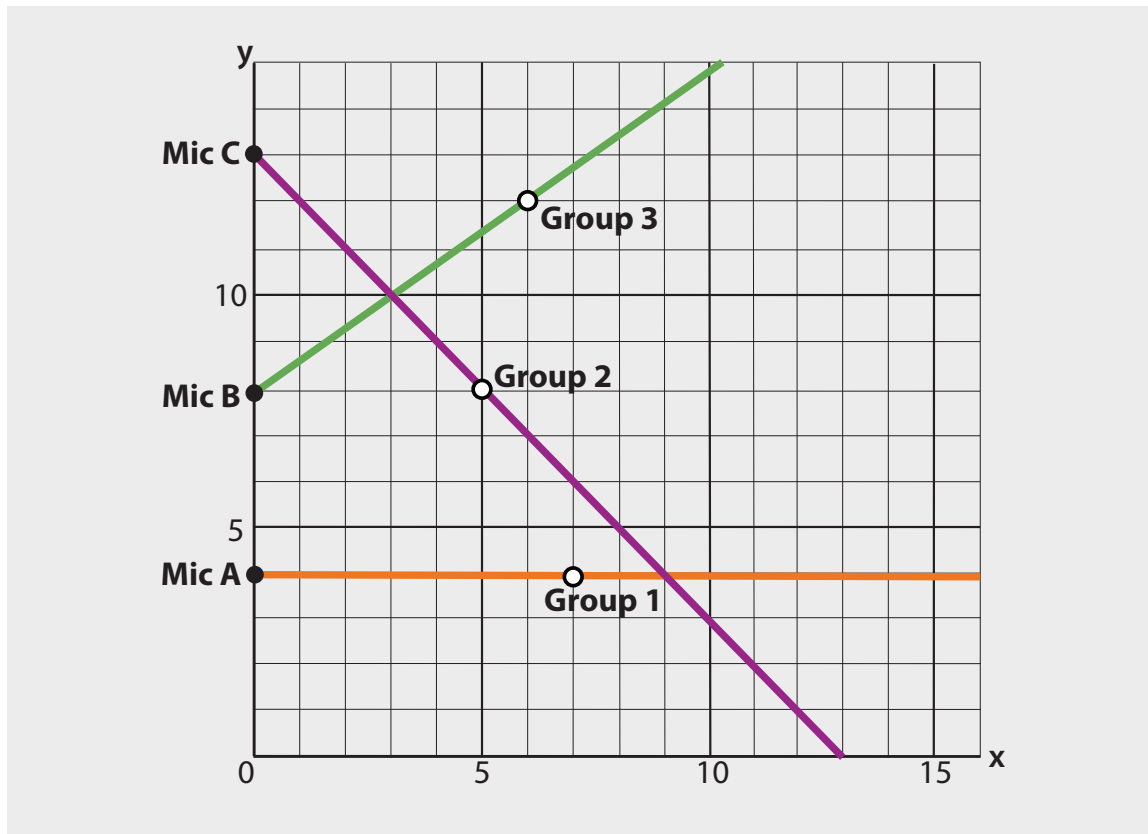
The Hawaiian Choir is split into three groups of three at the following points.

Group 1: (7, 4)

Group 2: (5, 8)

Group 3: (6, 12)

1. Plot these six points on a graph.



2.a. Microphone A is pointed directly at Group 1. Draw a line connecting these two points. What is the equation of the line that goes through these two points? (Use the point slope form: $y = mx + b$).

All of the microphones are on the y axis so it is easy to find the y intercept(s). The slope(s) can be found with the "rise over run" concept.

$$m = \frac{4 - 4}{7 - 0}$$

$$= \frac{0}{7}$$

$$= 0$$

$$b = 4$$

$$y = mx + b$$

$$= (0)x + (4)$$

$$y = 4$$

b. Microphone B is pointed directly at Group 3. Draw a line connecting these two points. What is the equation of the line that goes through these two points? (Use the point slope form: $y = mx + b$).

$$m = \frac{12 - 8}{6 - 0}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

$$b = 8$$

$$y = mx + b$$

$$= \left(\frac{2}{3}\right)x + (8)$$

$$y = \frac{2}{3}x + 8$$

c. Microphone C is pointed directly at Group 2. Draw a line connecting these two points. What is the equation of the line that goes through these two points? (Use the point slope form: $y = mx + b$).

$$\begin{aligned} m &= \frac{8 - 13}{5 - 0} \\ &= \frac{-5}{5} \\ &= -1 \\ b &= 13 \\ y &= mx + b \\ &= (-1)x + (13) \\ y &= -x + 13 \end{aligned}$$

3. Solo! One of the singers has to perform a solo.

a. Where could they stand in order for microphones A and C to record them clearly? Use your answers from Part 2, and the substitution method.

For microphones A and C to hear the soloist clearly, they need to stand where the microphone directional lines intersect. This is at (9, 4).

$$\begin{aligned} y &= 4 && \text{from part 2a} \\ y &= -x + 13 && \text{from part 2c} \\ (4) &= -x + 13 && \text{substitution method} \\ +x & && +x \\ x + 4 &= 13 \\ -4 & && -4 \\ x &= 9 \end{aligned}$$

b. Where could the soloist stand in order for microphones B and C to record them clearly? Use your answers from Part 2, and the elimination method.

For microphones B and C to hear the soloist clearly, they need to stand where the microphone directional lines intersect. This is at (3, 10).

$$\begin{array}{rcl}
 y & = & \frac{2}{3}x + 8 \quad \text{from part 2b} \\
 y & = & -x + 13 \quad \text{from part 2c} \\
 \hline
 y & = & \frac{2}{3}x + 8 \quad \text{elimination method} \\
 -(y) & = & -(-x + 13) \\
 \hline
 y & = & \frac{2}{3}x + 8 \quad \text{elimination method} \\
 -y & = & x - 13 \\
 \hline
 0 & = & \frac{5}{3}x - 5 \\
 +5 & & +5 \\
 5 & = & \frac{5}{3}x \\
 \times \frac{3}{5} & & \times \frac{3}{5} \\
 3 & = & x \\
 y & = & -(3) + 13 \quad \text{plug back into equation from part 2c} \\
 y & = & 10
 \end{array}$$

c. Does it make more sense to stand at the point from Part 3a or from Part 3b? Why?

This is open ended but most would agree that it makes more sense to stand at the point from part 3b, (3, 10), than the point from part 3a, (9, 4). This is because (9, 4) is behind the singing groups and it is very unusual for soloist to hide from the audience.

Activity 3.2 - Stage Setup II

Suppose that you set up four unidirectional microphones along the y-axis. The microphones are pointed left along the lines given by the following equations.

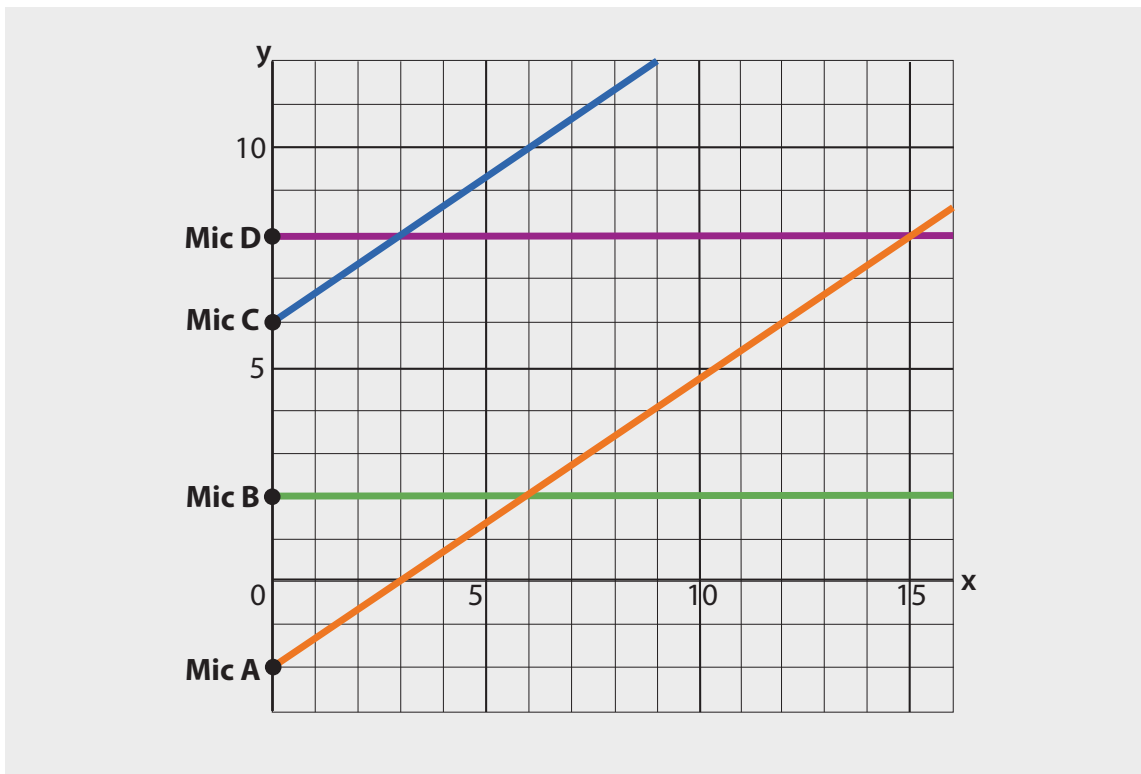
Mic A: $y = -2 + (2/3)x$

Mic B: $y = 2$

Mic C: $y = 6 + (2/3)x$

Mic D: $y = 8$

1. Graph the four equations for the mic directions. Label where the microphones are on the y-axis.



2. The soloist should stand where two microphones intersect. What is the coordinate where the soloist should stand? There is more than one possible answer, why did you choose to put the soloist here?

By looking at the graph, we can see there are three points where two microphones intersect. Students can choose any of these three points but must back up their choice with logic and articulate that logic. In these notes, we choose the microphone that is easiest to justify.

We know from life experiences that if we want to hear someone clearly, we should come reasonably close to them. The intersection of Mic C and Mic D are closest to the microphones so this point might be best suited for recording the voice of a single soloist. With the substitution method, we see that this point is at (3, 8).

$$\begin{aligned}y &= 8 && \text{Mic D equation} \\y &= 6 + \left(\frac{2}{3}\right)x && \text{Mic C equation} \\(8) &= 6 + \left(\frac{2}{3}\right)x && \text{substitution method} \\-6 & && -6 \\2 &= \left(\frac{2}{3}\right)x \\ \times \left(\frac{3}{2}\right) & && \times \left(\frac{3}{2}\right) \\3 &= x\end{aligned}$$

3. The backup singers should also stand where two microphones intersect, but away from the soloist. What are the coordinates where they should stand? There is more than one possible answer, why did you choose to put the backup singers here?

There are two more points left where two microphones intersect. The point that is closest to the microphones is taken by the soloist so let's choose the point that is the next closest. This is at the intersection of Mic A and Mic B. This point is at (6, 2).

$$\begin{array}{rcl}
 y & = & 2 \quad \text{Mic b equation} \\
 y & = & -2 + \left(\frac{2}{3}\right)x \quad \text{Mic A equation} \\
 (2) & = & -2 + \left(\frac{2}{3}\right)x \quad \text{substitution method} \\
 +2 & & +2 \\
 4 & = & \left(\frac{2}{3}\right)x \\
 \times \left(\frac{3}{2}\right) & & \times \left(\frac{3}{2}\right) \\
 6 & = & x
 \end{array}$$

4. There is a 'ukulele player standing at (3, 5). You have to set up one more microphone on the y-axis for their 'ukulele. This microphone is specially set up to record the sounds of a 'ukulele but microphones B and D are set up to record voices. What is the equation of the line that goes from the microphone to 'ukulele, but NEVER intersects microphones B or D?

Since the 'ukulele microphone must never intersect microphones B and D, all three microphones must have the same slope and different y intercepts. Both Mic B and Mic D have slope of $m = 0$. So our line equation becomes $y = mx + b = (0)x + b = b$. Now we need to find a y intercept, b that will cause the microphone to face the 'ukulele player at (3, 5). It is easy to see that $b = 5$ works. We can show this by replacing $x = 3$ and $y = 5$ (notice that we no longer have an x variable). So we have our equation $y = 5$.