Unit 5 Activities

Activity 5.1: Proof of the Pythagorean Theorem

Introduction:

Building materials can be hard to come by in the remote islands in the Pacific. Because of this, Pacific Islanders became really good at making the most of what they have. For example, people in Micronesia traditionally built larger canoes by stitching and tying smaller pieces of wood together. Also, Hawaiians weaved small leaves together to make huge floor mats and canoe sails. In this activity, we will learn how to make a big square using any two small squares, and we'll do this without wasting any material. We'll also learn more about the Pythagorean Theorem.

Materials needed:

1. 2 sheets of paper, square shaped, different sizes
2. Pencil or pen
3. Ruler or straight edge
4. Tape
5. Scissors

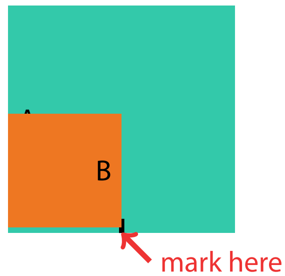
Activity

The Pythagorean Theorem tells us if a right triangle has sides of length A, B, and C, where C is the length of the hypotenuse, then $A^2 + B^2 = C^2$. In this activity we will prove it.

1. First we will make two squares. One will have sides of length A and area A^2, and the other will have sides of length B and area B^2.
2. Then we will make a right triangle with sides of length A, B, and C, where C is the length of the hypotenuse.
3. Next we will cut the two squares and tape them back together to make one big square with area C^2.

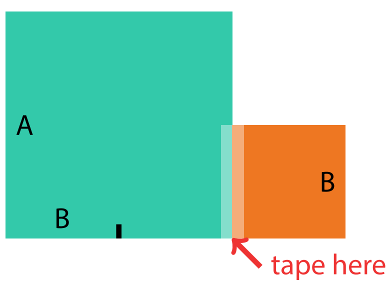
Step 1: Cut out two squares. They don't have to be different sizes but this activity is more interesting if they are.	**Step 2:** Label the left side of the bigger piece A and the right side of the smaller piece B.
	

Step 3: Use the smaller square to make a mark on the bottom of the bigger square that is B away from the left side.

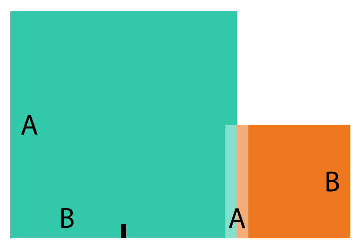


Step 4: Label this distance B.

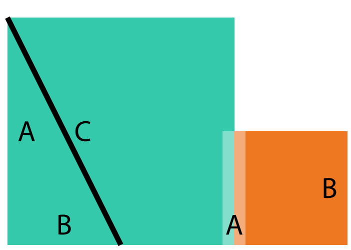
Step 5: Tape the left side of the smaller square to the bottom of the right side of the bigger square.



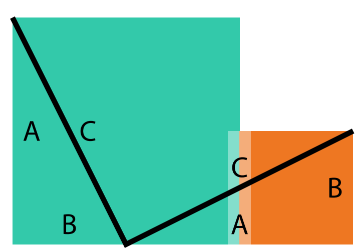
Step 6: Label the distance between the marker and the right side of the small square A. Talk to your friends to make sure everyone knows why this distance is A.

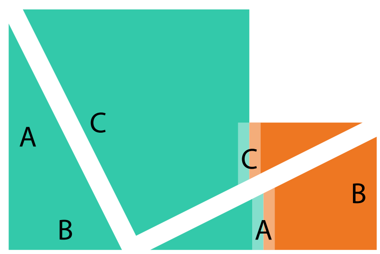
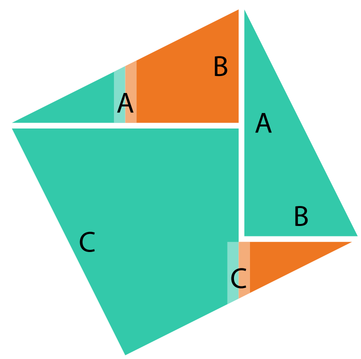


Step 7: The edges A and B on the left side are perpendicular so we can use the ruler to draw a hypotenuse C.



Step 8: The edges A and B on the left side are also perpendicular so we can use the ruler to draw another hypotenuse C. Talk to your friends to make sure everyone knows why this C is the same as the last one.



<table>
<tr><td>Step 9: Now let's cut along the hypotenuses to get two right triangles.</td><td>Step 10: Use translations, rotations, and/or reflections to move the right triangles to make a large square. There are many ways to do this.</td></tr>
<tr><td>
</td><td>
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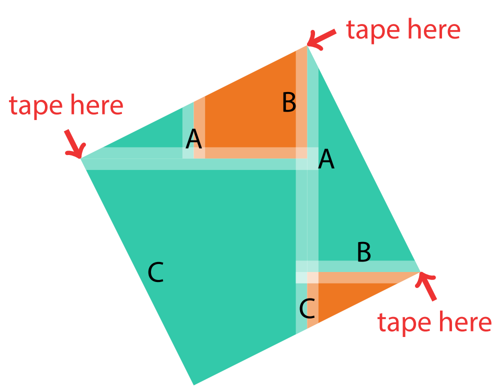
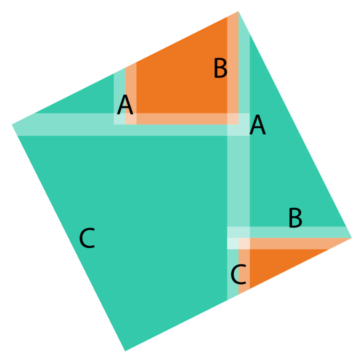
<tr><td>Step 11: Tape the pieces down.</td><td>Step 12: We're done! You started with two squares of areas A^2 and B^2.</td></tr>
<tr><td>
</td><td>
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What is the area of this final square? How is this area related to the areas of the original squares?

Activity 5.2: Pythagorean ʻUpena (Net)

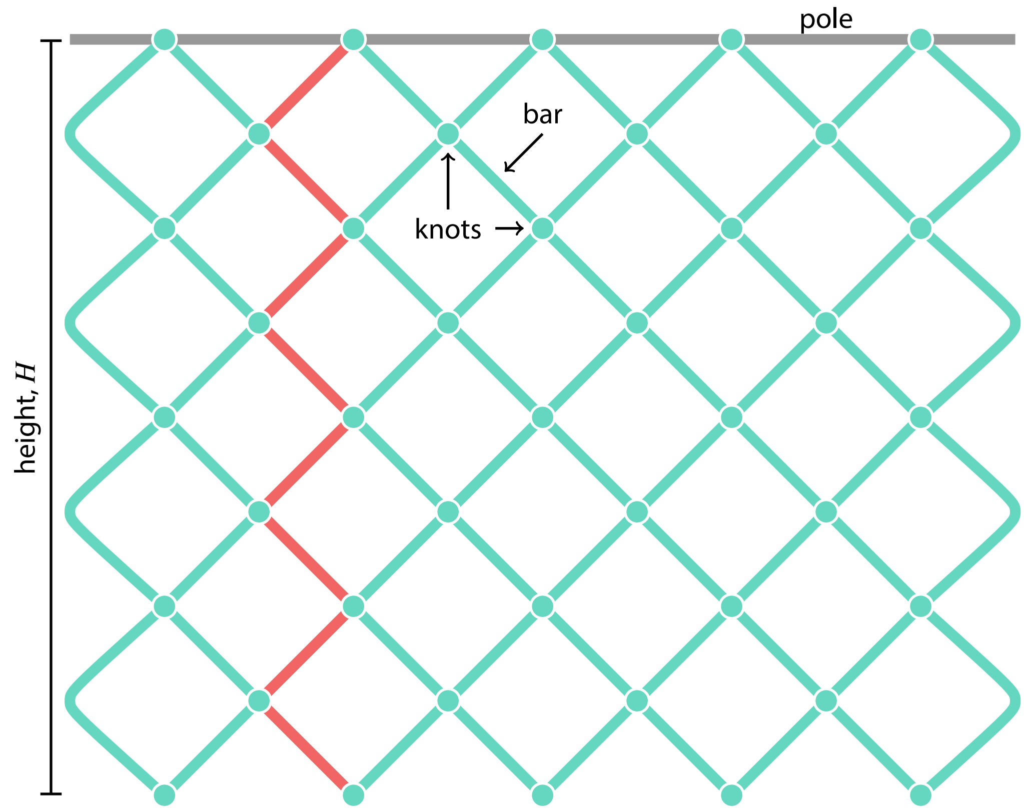
Materials needed:

1. String
2. Scissors
3. Ruler
4. Plastic pole or a long smooth stick

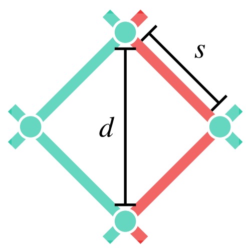
Activity

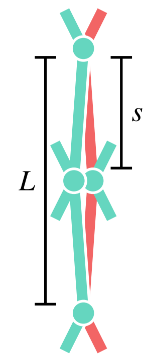
Kuʻulei is making a net. She knows how tall she wants her net to be and how big each diamond-shaped hole should be. She doesn't want to waste any string or make the knots uneven. With a neighbor or a group, let's use our knowledge of triangles and irrational numbers to help her plan out her net design.

In the picture below, we see that the net is hung from a pole. This makes it easier to sew/tie the net together. The net is made up of strings with their centers tied to the pole. *Half* of a single string is shown in the diagram in red. The net is made up of **knots** and **bars**. All bars are of equal length, s, and the height of the net is H.



Even when you know the length of the bars, calculating the height can be harder than you think. When the net is light and empty, the holes are wide open and the height of each hole is the diagonal d. When the net has a lot of weights on the bottom and is full of fish, then two corners of each hole will stretch until the hole closes. The height of the hole then becomes what's called the "eye length", L.





1. The following equations are true. Talk to your neighbor and explain why.

$$\text{Light net:} \quad d = \sqrt{2} \times s$$
$$\text{Heavy net:} \quad L = 2 \times s$$

You can write down the explanation here if you want. It'll help make the rest of this activity easier.

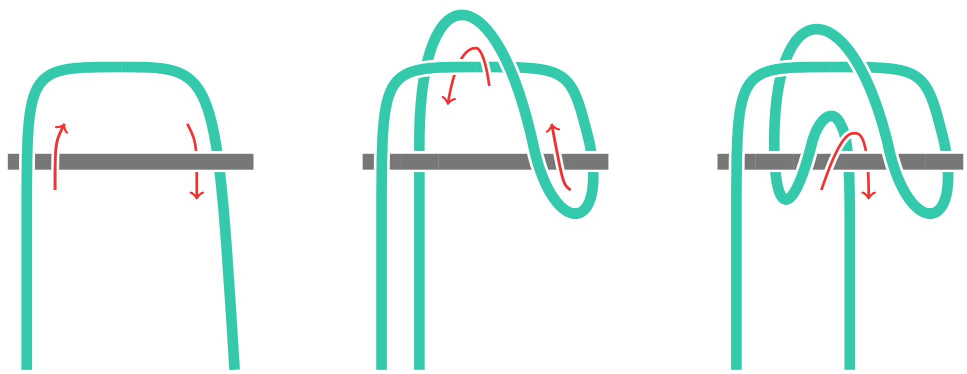
2. So the height of each hole is at least d and at most L. If we have n holes from the pole straight down to the bottom of the net, then our height is at least $d \times n$, and at most $L \times n$.

Talk to your neighbor and explain why the following inequalities are true.

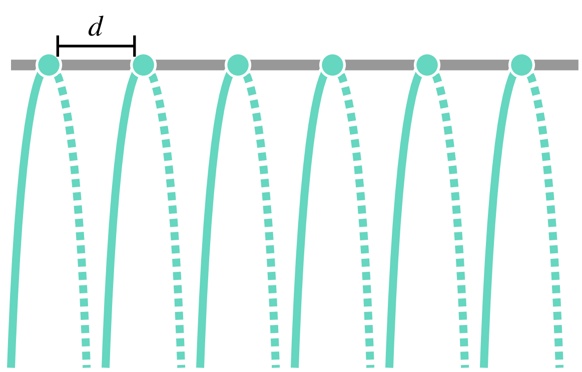
$$H \geq \sqrt{2} \times s \times n$$
$$H \leq 2 \times s \times n$$

3. Kuʻulei wants a net that is 5 feet in height. She also wants the sides of the holes to be 3 inches. What is an okay number of holes n from top to bottom? (There are more than one possible answer).

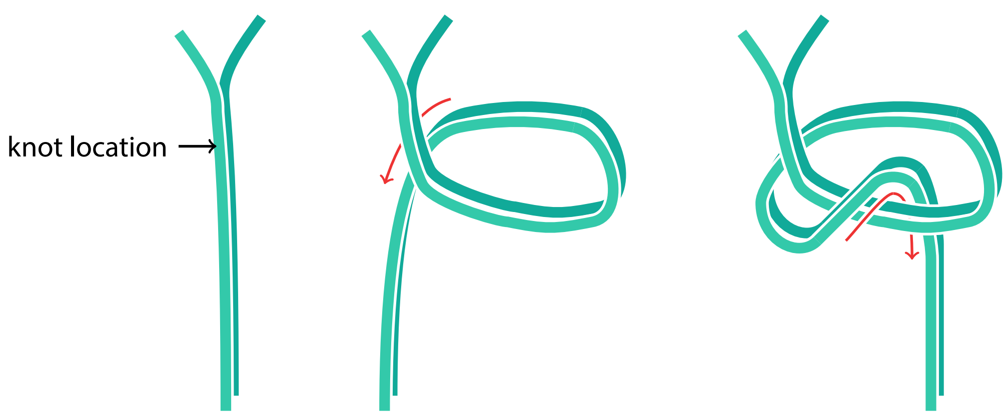
4. Now that we've helped Kuʻulei design her net, let's use what we learned to make our own net. To begin, let's get a smooth stick or pole and cut some pieces of string to tie to it. We'll leave the calculations up to you to decide how long each piece should be. Once you choose values for H and s, then you can calculate everything else. The following diagram shows us how to tie each string to the pole.



Please tie this knot in the center of each string and space them a distance of d apart on the pole. Remember that d is the length of the diagonal of the holes you want.



Now we can grab pairs of strings and knot them together to make the net! Here's how we knot two neighboring strings together.



Continue to tie the strings one row at a time. Make sure to take careful measurements as you go so that the net will come together nicely. It’s up to you whether you want to measure the holes or the strings!

When you are pau (done) with this activity, slide the knots off of the pole to get your completed net!